Grade 10 Maths

UNIT- 6

PLANE GEOMETRY TEOREMS OF A TRIANGLES

⇒Three or more points that lie on on line are called collinear points
⇒Three or more lines that pass through one point are called concurrent lines.

L

P Q R S T

Collinear points

4

Fig 1

Concurrent lines

⇒Theorems about col linear points and concurrent lines are called incidence theorems.
⇒A line that devides an angle in to two congruent angles is called an angle is called an angle bisector of the angle.
⇒A line segments that divides a line segment in to two congruent line segments is called bisector of line segments.

B

A p

A CFig3 AO≡OP

D

M(<ABD)=M(<DBC) L

∴ $\overline{DB}$ is called an angle bisector ∴ a line L is called bisector of a line segment

⇒When a bisector of a line segments forms right angle with the line segment then it is called the perpendicular bisector of the line segments. See from fig.3

<u>Median of a triangle</u>

⇒A median an of a triangle is a line segment drawn from an vertex to the mid-point of the opposite side.

B B B

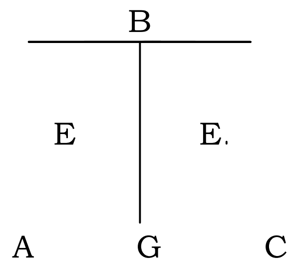
E F

A C A

D C A C

→$\overline{BD}$ is the median →$\overline{AE}$ is the median →$\overline{CF}$ is the median

Theorem 6.1 (thrm)the medians of a rectangle are concurrent at a point $\frac{2}{3}$ of the distance from each vertex to the mid point of the opposite side .



Note

1. AO= $\frac{2}{3}$(AE) $\Rightarrow$ AO+OE=AE

2. OE=$\frac{1}{2}$(AE)$\Rightarrow$ $\frac{2}{3}+\frac{1}{3}$=1 full of AE

3. BO=$\frac{3}{2}$(BG) $\Rightarrow$ BO+OG=BG

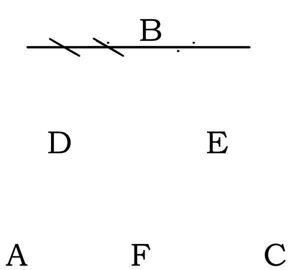
4. OG=$\frac{3}{2}$(BG) $\Rightarrow$ $\frac{2}{3}$ +$\frac{1}{3}$=1 full of BG

5. CO=$\frac{2}{3}$(CE) $\Rightarrow$ CO+OE=CE

6. OE=$\frac{1}{2}$(CE)$\Rightarrow$ $\frac{2}{3}+\frac{1}{3}$=1 full of CE

Example solve the foll.

From the following figure



If $\overline{AE}$,$\overline{BF}$ and CD are medians of a triangle ABC, And

M($\overline{AE}$) =24cm

M($\overline{OF}$) =8cm

M($\overline{CO}$) =18cm

a.Then find the length of : a, CD b, OE

Soln : (a) (b) AO=$\frac{2}{3}$(AE) (c) OD

CO =$\frac{2}{3}$CD OE=$\frac{1}{3}$(AE) CO=$\frac{1}{3}$(CD) m.b.s by $\frac{3}{2}$ we get

$\frac{3}{2}(CO) = \frac{3}{2}(\frac{2}{3})CD$

$\frac{3}{2}CO = CD$

$CD = \frac{3}{2}(10cm)$

$= \frac{30}{2}cm$

$=15cm$

$= \frac{1}{3}(24cm)$

$= \frac{24}{3}cm$

$=$ <u>8cm</u>

$= \frac{1}{3}(15cm)$

$= \frac{15}{3}$ cm

$=$ <u>5cm</u>

The point of intersection of the medians of the triangle is called the centroid of the triangle

Altitude of a triangle

- The altitude of a triangle is a line segment drawn from a vertex perpendicular to the opposite or to the opposite side produced.

- AD and CE are

The altitudes of the $\triangle ABC$

Question

Write the difference between the altitude and median of the given triangle.

- the perpendicular bisectors of the sides of any triangle are concurrent at a point which is equidistance from the vertices of the triangle

AE, BF and CD are the perpendicular bisectors of a ABC

Hence m(AP)=m(BP)=m(CP)

Conceptually If m(AP)=5cm, then m(BP)=5cm

- The point of intersection of the bisectors of the angles of the triangle is called the incentre of the triangle.

Altitude theorem

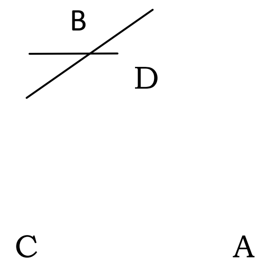
- the altitude theorem stated discused there is or only right angled triangle. It relates the length of the altitude to the hypenuse of a right angled triangle,to the lengths of the segments of the hypotenuse.

- the altitude theorem states as:

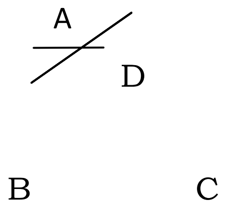
In right angle triangle ABC with altitude CD to the hypotenuse AB: <u>AD</u>=<u>CD</u>

DC DB

⇨ $(AD)(DB)=(CD)^2$



Example



If BD=4cm and CD 8cm calculate AD

sol^{n} $(CD)(AD)=(BD)^2$

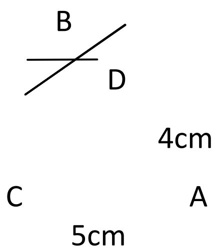
$(8CM)\ (AD)=(4CM)^2$

$8ADcm=16cm^2$

$\frac{8ADCM}{8CM}=\frac{16CM^2}{8CM}=2CM$

Exercise from the following right angle triangle. If AC=5cm,AD=4CM, then find

a)BD b) BC



Sol^{n} from△ADC, since <D is right angle then△ADC is right angled triangle. Then

$(CD)2+(DE)2=(AC)^2$ a) $(AD)(DB)=(CD)^2$ b) $(BC)^2+(CA)^2=(BA)^2$

$CD^2=25CM^2-16CM^2$ $(4CM)BD=(3CM)^2$ $(BC)^2=(BA)^2=(CA)^2$

$CD^2=9CM^2-6CM^2-16CM^2$ $(4CM)BD=(3CM)$ $(BC)^2=(BA)^2-(CA)^2$

$CD^2 = 9cm^2$ $\frac{4DBcm}{4cm}=\frac{9cm}{4cm}$ BC= $\sqrt{(4+9/4)^2-25}$

$CD=\sqrt{9cm}$ BC= $\sqrt{25/4)^2-25}$

CD=3cm DB=9/4 cm =7.5cm

Special quadrilateral

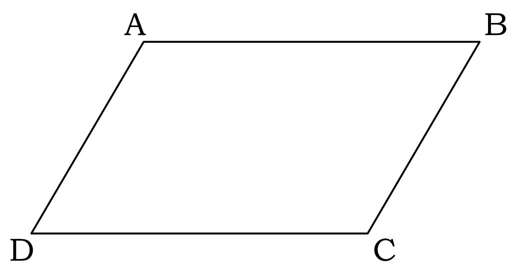
⇨ The special quadrilateral are 1. Trapezium 2. parallelogram 3. rectangle 4. rhombus 5. square

1. **Trapezium:** a trapezium is a quadrilateral where only two o the sides are parallel.

- $\overline{AB}$ and $\overline{DC}$ are bases of the trapezium
- $\overline{AD}$ and $\overline{BC}$ are called are called the legs of the trapezium
- $\overline{AE}$ is called he height of the trapezium

⇨ If $\overline{AD}$ and $\overline{BC}$ are congruent (equal) then it is isosceles trapezium.

2. **Parallelogram:** is e quadrilateral lateral in which both pairs of opposite sides are parallel.



Exercise write the six (6) properties f parallelograms (a-f) page (241)

3.Rectangle: A rectangle is a parallelogram in which one of its angle is right angle (90^0) ie all of the angles are measures 90^0 each ie $190+90+90+90^0)=360^0$

- All rectangles are parallelogram (True/False)
- All parallelograms are rectangle (True/False)
- Draw the figure of rectangle and write the basic properties of a rectangle.

4.Rhombus: A rhombus is a parallelogram which has two congruent adjacent sides.

- try to sketch the figure of rhombus and write the basic properties of the rhombus
- All rhombus as are square (True/False)__________
- All squares are square (True/False)

5. **Square:** A square is a rectangle which has congruent adjacent sides.

- square has the properties of a rectangle.
- square has all the properties of a rhombus.

More on circle

O' is called cenire of the circle

AB is called Diametre of circle

DC is called the radius of atrcircle

AxC is called an are

Note 1 diameter=2radius (1D=2r ⇨ r=1/2 D

- **An inscribed angle:** an angle whose vertex lies on the circle and whose sides are chords of the circle.

A

M(<ABC)=$\frac{1}{2}$(AxC) ●O B

C

- The measures of an angle inscribed in acircle is half the measure of the subtending it.

1. An angle inscribed in a semi-circle is a right angle (90^0)
2. A sem circle is obtuse
3. An angle inscribed in an arc greater than a cemi-circle is accute.
4. An angle formed by a tangent and chord drawn from the point of the tangency is measured by half the are it intercepts.
5. The measures of an angle formed by two chords inter secting in side a circle is half the sum of the measures of the arc subteding the angle and its vertically opposite angle.

~~1.~~____ 2.____ 3.____ 4.____ 5.____

m(ABC)	m(<ABC)	m(<ABC)	m(<ABC)	m(<APC=1/2(AxC+BYD)
=1/2(AxB)	=1/2 (AYC)	=1/2 (AxC)	=1/2(ADC)	=1/2(AxC)+ 1/2(BYD)
=1/2 (180)	=1/2(>180)	=1/2 (<180)		=1/2 (AxC+BYD)
= 90^0	⇨90 ⇨	<90.		
	b/n 90 &180	Which is accute angle		
	W/c is obtuse			

Example see the foll circle if m(<BOD) =30 and m(CSD)=40 ,find m(ATC).

Soln m(BOD) =m(<AOC) =1/2(ATC) + ($\overline{BCD}$)

Hence m(<BOD)=1/2(BSD +ATC)

⇨ 2(30)=40 +ATC

..M(ATC)=2(30)-40

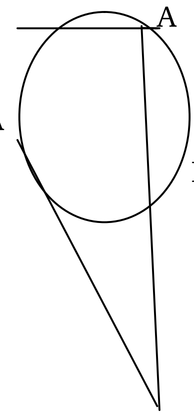
M(ATC)=60-40=20

1. The measures of the angle formed by the lines two chords intersecting out side a circle is half the difference of the measure of the arcs they intercept .
2. The measure of an angle formed by a tangent and a secant drown to a circle from a point out side a circle is equal to half the difference of the measures of the intercepted arcs.
3. A secant and tangent are drown from a point out side a circle ,then the square of the length of the tangent is equal to the product of the length of line segments.

1. A B

C Y D

P



A

a

C

P b B

M(<p)=1/2 (m(AxB-CJD)

=1/2(m)(AxB-CYD)

1/2m(AxB-CYD)

m(<P)=1/2m(AxB-BD)

=1/2m(AxB-BYD)

$(PA)^2=(PB)(PC)$

$a^2-(b^2)(b+c)^2$

∵ $(PA)^2=(PB)(PC)$

Example 1. ***2.*** ***3. A***

6CM P ***X*** ***Y*** ***P***

Q ***R***

C

If PQ=12cm

RQ=8cm

find PR

Hint $(PT)^2=(PR)(PQ)$

and 4.5cm

If m(<p)=5^0

and m(AxB)=40^0

find m(CYD)

If m(APC)=30^0

m(<AFC)=85^0

find AC, m(BD) and m(<ABC)

Regular Polygons

- A polygon whose vertices are on a circle is said to be inscribed in a circle the circle is circumscribed about the polygon.

$\Rightarrow$ A polygon whose sides are tangent to acircle is said to be circumscribed about the circle.

$\Rightarrow$ For any n-sided regular polygon.

a) Appothem $a = r\cos\left(\frac{180^0}{n}\right)$

b)Side of length $(s) = 2r\sin\frac{180^0}{n}$

c) Perimetre: $p = ns = 2nr\sin\frac{180^0}{n}$

d)Area $(A) = \frac{1}{2}ap = r\cos\frac{180^0}{n}$

$A = \frac{1}{2}nr^2\sin\frac{180^0}{n}$

Note r=radius n=number of sides

a=apothem A=Area

Exercise find a)apothem b)length of sides(s) c) perimetre(p) and Area (A) of a regular quadrilateral whose length of radius r=5cm.